rating method to a series of parametrically selected mode interaction cases, separation boundaries can be obtained. Simulation of the analytical cases would then follow to verify the predicted pilot ratings. This work is in progress at Oklahoma State University.

Acknowledgment

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High-Altitude Long-Range Sonic Boom Propagation

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I. Introduction

THIS Note investigates the magnitude and importance of on-track secondary sonic booms, which are propagated upward from supersonic aircraft, and then reflected back down to the Earth's surface by gradients in sound propagation velocity in the atmosphere, as sketched in Fig. 1. The results of the approximate analysis and computations show that the on-track secondary booms are extremely weak, except close to the aircraft, and are not likely to be of any importance to observers on the ground or to measurably affect the Earth's atmosphere.

The sonic boom under the track of an aircraft such as the Concorde will typically consist of N-shaped signatures of order 100 N/m^2 amplitude. Signals from the Concorde which initially propagated upward and to the side but were then reflected down have also been measured by Balachandran, Donn, and Rind.[1] These signatures are typically below 2 N/m^2 in amplitude. However, after various reports of audible and infrasonic disturbances on the East Coast of the U.S. beginning in December 1977, some speculation arose that the reflection of upward-traveling booms from the thermosphere might be the cause of some of these events, and that significant changes in the upper atmosphere might be caused by the upward-propagating sonic booms from supersonic aircraft.[2,3]

This Note reports on the analysis and prediction of on-track booms propagated initially upward from a supersonic aircraft. On-track booms are those which propagate in the vertical plane through the flight path. Numerical results are presented for numbers representative of a maximum weight Concorde at cruise conditions. The atmosphere is assumed quiescent and stratified but with properties homogeneous in planes parallel to the Earth's surface assumed locally flat.

The analysis and calculations follow Ref. 4. Given an initial perturbation pressure δp at altitude H, the pressure at a new location is found in terms of properties at the new altitude from $(\delta p)^2 A/\rho c = \text{const.}$, where A is the ray tube area and ρ the density. The nonlinear advance in time of any portion of the wave compared to an infinitesimal acoustic wave is[4]:

$$\Delta t = \left(\frac{\delta p}{p}\right)_H \frac{\gamma+1}{2\gamma a_H} \int_{r_H}^{r} \frac{p_H}{p} \left(\frac{\rho a_H A_H}{\rho_H a A}\right)^{1/2} \frac{M}{\beta}\, dr$$

Due to the increase in the integrand with altitude, Δt is very large for upward-propagating sonic boom waves leading to very strong nonlinear dissipative effects. On the other hand, downward-propagating waves in the atmosphere are much less affected by nonlinearity.

Once the advance Δt has been found, δp vs t is constructed at a given point and then each δp point is advanced (or retarded for $\delta p < 0$) and then shocks are introduced, balancing the cutoff areas of the multivalued advanced δp vs t curve, as discussed in Ref. 4. In the present analysis, this was carried out analytically in the computer program for assumed asymptotic (triangular) wave shapes, except for one case carried out graphically for the logarithmically singular wave shape resulting for an idealized linear caustic, as discussed later.

The energy carried by a wave can be found from the acoustic intensity (power/area/time) which is $(\delta p)^2/(\rho c)$. Integrating over the wave period, one finds the energy/area carried by the wave as $E/A = \int (\delta p)^2/(\rho c)\, dt$. Assuming a triangular wave with amplitude Δp and length T (half of an N-wave), one obtains $E/A = (\Delta p)^3 T/3\rho c$. The impulse is given by $I = \int_0^T \delta p\, dt$. At the ground, Δp and I are multiplied by a ground reflection coefficient of approximately two.

II. Estimated Behavior in the Caustic Region

The rays from the aircraft will become horizontal and turn downward at a caustic at the altitude where the speed of sound is equal to the aircraft speed. This occurs at 160.86 km in the U.S. Standard Atmosphere, 1976, for a Concorde flying at $M=2$, where the speed of sound is 295.03 m/s. The low density at 160 km results in a mean free path of approximately

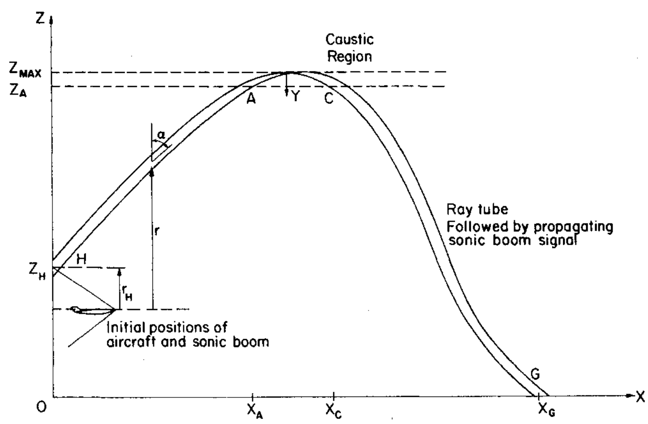

Fig. 1 Sketch of upward ray tube trajectory and definition of symbols.

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50 m. As the shock waves in the N-wave are weak, the shock thicknesses can be estimated to be of order 10^3 m. This is quite large, but still less than the signature half-length of order 2×10^3 m. Thus, the concept of the shock being treated separately from the rest of the signature is not unreasonable. However, the results must be considered as estimates rather than exact calculations.

If the large mean free path and viscous effects are neglected, there are still two other difficulties with geometrical acoustics in the caustic regions. Linear turning point and nonlinear effects both occur and must be considered. For waves which are short compared to the ray curvature, the inviscid linear problem can be treated as a turning point, resulting in local solution in terms of Airy functions. Fourier components of an entering signal leave the caustic with a phase advance of one-quarter wavelength and this has been shown by Hayes[5] to transform an incoming N-wave given by:

$$\delta p = -\Delta p t[H(t+T) - H(t-T)]$$

into an outgoing signal at an equal ray tube area of

$$\delta p = -\Delta p \frac{t}{\pi} \ell n \left| \frac{t-T}{t+T} \right| - \frac{2T}{\pi}$$

as shown in Fig. 2. This figure also shows how nonlinear distortion and shock effects modify this signal after the caustic (see discussion above and Ref. 4).

In order to find the asymptotic shock resulting from the nonlinear advance of this logarithmic wave shape, the value of the area under the positive peak is required. Using that $\int_{-\infty}^{+\infty} \delta p dt = 0$ and symmetry, the outgoing signal was integrated numerically to give $0.3819\,\Delta p_A T_A$, compared to $\frac{1}{2}\Delta p_A T_A$ for the incoming wave. Thus, a low estimate of the area of the leading part of wave leaving the caustic can be taken as $0.3819\,\Delta p_A T_A$. If the wave is assumed to eventually approximate its asymptotic nonlinear form of an N-wave again, one can use an outgoing linear equivalent N-wave of $T_c = T_A$, $\Delta p_c = 2(0.3819)\,\Delta p_A$. For the most conservative estimate, we assume $\Delta p_c = \Delta p_A$.

The actual behavior at the inviscid caustic is nonlinear and quite complex. The local behavior could be best described using the similarity transformations of Guiraud[6] and Hayes[7] along with the numerical results of Gill and Seebass,[8] as discussed by Plotkin and Cantril.[9] For the present calculations, estimates based on Plotkin's presentation show that the nonlinear region of the caustic is only about 200 m thick, and that the maximum C_p expected would be on the order of 0.08. However, the region is approximately 8 km long in the x direction and can include some significant nonlinear advance of the signal. For our low-estimate approach, we analytically approximated the advance between

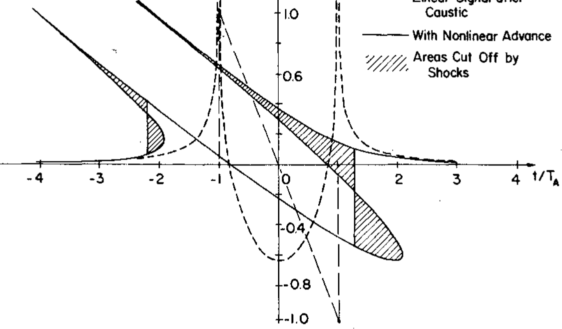

Fig. 2 Incoming N-wave, linear signal after inviscid caustic, and ground level nonlinear signal with shocks.

our assumed incoming and outgoing points, assuming $C_p Y^{1/4} = \text{const}$, which would only hold for a linear inviscid caustic. The advance is then convergent, being proportional to $Y^{3/4}$. We then used this additional advance to modify the equivalent outgoing triangular wave shape before computing the propagation downward to the ground by geometrical acoustics. For our conservative estimate, we assume $\Delta p_c = \Delta p_A$ with no additional advance in the caustic region. Either approach results in an output signal from the caustic region at 160 km altitude, but displaced approximately 36 km horizontally from the point A following the locally circular rays. This significant X displacement from the incoming to the outgoing ray tube required the correction of the ray tube areas and pressures coming out of the caustic for the divergence of the rays in horizontal planes as they passed through the caustic region.

III. Results and Discussion

The input data assumed were obtained from the work of H. W. Carlson.[10,11] The Concorde maximum takeoff weight of 185,000 kg was assumed with $M=2$ cruise at 17 km altitude. (The numerical results are very insensitive to aircraft weight.) The aircraft length was taken as 62.1 m and the U.S. Standard Atmosphere, 1976 was assumed. Using these numbers, the sonic boom, 1 km above the aircraft (at 18 km), was estimated to be equivalent to an N-wave of $\Delta p_H = 22.6$ N/m^2 and $T_H = 0.0526$ s.

At 160 km altitude the calculated upward-propagating N-wave has a strength of $\Delta p_A = 0.978\times10^{-5}$ N/m^2 and a half period of $T_A = 3.18$ s. The advance from 18 to 160 km has the very large value of 192 s, showing the extreme importance of the nonlinear shock effects and justifying the use of an N-wave as input. The nonlinear dissipation reduces the linear pressure perturbation by a factor of nearly 10^{-2} by 160 km. The pressure coefficient at 160 km is only 0.046, showing that only negligible shock heating effects will occur.

Another way of considering possible heating effects is by looking at the energy associated with the wave. The energy per unit area carried radially by the front half of the N-wave (double this for the full wave) is given by $(E\cos\alpha)/A = (\delta p)^2 \cos\alpha/3\rho c$ where α is the ray angle to the vertical. This energy measure was computed to vary from 2.15 j/m^2 at 18 km, to 1.13×10^{-1} j/m^2 at 20 km, to 5.92×10^{-4} j/m^2 at 80 km to 1.69×10^{-5} j/m^2 160 km. This clearly shows the very small energy per unit area radiated to high altitudes. If these values are multiplied by r, a measure of area, the results show that approximately 85% of the radiated energy left at 1 km from the aircraft is dissipated within 3 km of the aircraft and 98% of it is dissipated below 80 km. Even accounting for the low density at high altitudes, the temperature rises at all altitudes are negligible fractions of a degree. Thus, no measurable thermospheric heating will occur. Similarly, the velocities and momentum associated with the weak N-waves are also negligible.

The calculated order-of-magnitude results for disturbances at the ground for the "Concorde" case are given in Table 1. The first two estimates assume the asymptotic N-wave approximation. More accurate estimates found using a graphical advance and shock construction based on a linearized caustic are also shown. The last line gives rough estimates of the primary shock boom of the Concorde for comparison.

The x locations of the various ray positions were also calculated. The initially upward ray, after reflection, intercepts the ground at 357 km ahead of the aircraft location where it was originally generated.

It is interesting to compare these computed order-of-magnitude disturbance magnitudes to those measured by Balachandran et al.[1] The disturbances they measured originated from rays intermediate between the primary boom and the upward on-track rays calculated here. Thus his Δp's

Table 1 **Properties of Concorde on-track sonic boom signal after reflection by the thermosphere compared to the primary boom**

	$2\Delta p_G$,N/m^2	T_G,s	$(E/A)_G$,j/m^2	$2I_G$,NS/m^2
Conservative estimate	0.048	6.97	3.24×10^{-6}	1.68×10^{-1}
Low estimate	0.036	7.08	1.86×10^{-6}	1.28×10^{-1}
Linearized caustic, graphical advance	front 0.020	4.5	—	—
and shock	rear 0.036	6.6	—	—
Primary boom below Concorde (approximate)	75	0.12	1.35×10^{-1}	7.5

of order 2 N/m^2 and half-periods (T) of 0.5-3.5 s are intermediate between the primary on-track results and the present calculations. The present approach could be modified to deal with intermediate rays, with the major complications being in the ray tube area analysis and in accounting for a stratified wind structure in the atmosphere.

To put the present order-of-magnitude calculations into perspective, comparison with the primary boom of the Concorde shows that all signature measures are negligible, except perhaps for the impulse which is as great as 2.2×10^{-2} of the Concorde value due to the increased T. This impulse is still very small, but perhaps could lead to some structural vibration of buildings with very low resonant frequencies. However, even if fully coupled, with vibration amplitude squared taken to be proportional to energy, the vibration energy squared ratio would be proportional to the E/A ratio or about 46 dB below the Concorde's primary boom value. Of course, individual occurrences will vary statistically around these values due to atmospheric inhomogeneities and nonuniform flight conditions. These might result in amplifications of order 5 at most,[12,13] but even with such amplifications, it appears extremely unlikely that the on-track secondary sonic boom can be involved in any reported audible events.

Similar general conclusions have been recently reached independently in Ref. 14.

Acknowledgment

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Technical Comments

Comment on "Flight Test of Stick Force Stability in Attitude-Stabilized Aircraft"

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A PAPER by Mooij and van Gool[1] reported results of flight tests which investigated the effects of positive stick force stability (PSFS) on the handling qualities of an airplane equipped with a pitch-rate-command/attitude-hold (PRC/AH) longitudinal control system. It was found that glide path control deteriorated at the highest value of PSFS gradient investigated.

The increase in PSFS gradient during the investigation reported in Ref. 1 was accompanied by reduction in the damping of the long period (phugoid) longitudinal oscillation. In fact, at the higher value of this gradient the phugoid motion (or its equivalent with the "nonaerodynamic" control system used on the airplane) was an unstable oscillation. This was attributed to the value of the stability derivative M_u which was introduced by the mechanization of the PSFS control system used in this investigation.

It is this writer's opinion that the observed deterioration of glide path control may well have been caused by the destabilization of the phugoid mode rather than by the introduction of positive stick force stability. Flight research performed more than twenty years ago[2] produced results

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